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Propagation of SV waves in a periodically layered media in nonlocal elasticity

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Abstract

Effect of nonlocality on the dynamic behavior of laminated composites is investigated by means of dispersion of vertically polarized harmonic shear waves propagating in the direction parallel to layering. Nonlocal elasticity is briefly summarized and the dispersion relation for symmetric and anti-symmetric waves are obtained within the framework of both classical and elasticity. The numerical structure of the problem is investigated as to reduce some possible incorrect results which may take place in the solution. Results are displayed in a serious of figures. Advantages of nonlocal elasticity in representing the mechanical behavior of composite materials is discussed.

Keywords: Nonlocal elasticity; Wave propagation; Composites; Composite materials; Theory of elasticity

1. Introduction

Laminated composites have been becoming increasingly important in various branches of modern technology because they offer more attractive and cost effective solutions for a large variety of problems. For example, in structural design, laminated composites find increasing applications not only because they have high strength-to-weight and high stiffness-to-weight ratio but also they are more resistive to environmental effects and they require less maintenance.

Analysis of mechanical behaviour of composite materials plays a central role in various stages of composite technology. There is a vast amount of literature on the analysis of mechanical behavior of laminated composite materials. Among them Basar et al. (1993) gave an extensive model for the static behavior of laminated composites, a unified derivation of various shear deformation models with ability to deal with finite rotations. Finite element formulation of their equations is also included in this study as well as a comparative numerical study demonstrating the prediction capability of different models of analysis of static behavior of laminated composites.

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Dynamic behavior of composite materials is as important as static ones, in many cases more important. But, the models developed for analysis of static behavior of composite materials are not suitable to represent most of the features of dynamic behaviors. For example, they all are far from predicting dispersion relation for waves propagating in a composite material. But it is an experimental fact (actually, it would not be difficult to see by a mind experiment as well) wave propagation is highly dispersive. On the other hand, dispersion of waves propagating in an heterogeneous medium has attracted the attention of scientists since more than five decades because of the importance of the seismic waves and the nature of the seismograms recorded from earthquakes. First systematic treatment of this problem was given by Postma (1955) which the earth was considered as a layered heterogeneous medium and was modelled by an transversely isotropic one. This approach (also called "effective modulus theories") has later been adopted for the analysis of layered composites with successful results for static and quasi-static problems (for example see, Christensen, 1979; Nemat-Nasser and Hori, 1993). But as far as the dynamic problems are concerned, it has been observed that this method is insufficient in predicting many aspects of the dynamic behavior of materials. That is why the dynamic problems, especially the ones related to wave propagation, are studied separately.

The fact that the propagation of wave in a nonhomogeneous medium is dispersive, motivated researchers to construct adequate models for the dynamic behavior of layered composites. (Achenbach et al., 1968; Sun et al., 1967; Sun et al., 1968a; Herrmann and Achenbach, 1967, 1968) proposed "the effective stiffness theory" where the laminae are considered as plates for which the displacements are taken to be linearly dependent on the distance along the thickness. This approach which suffers from the discontinuity of the stresses at the interface was refined Drumheller and Bedford (1973) by adding a quadratic term of the variable along the thickness to the displacement. Another major approach to the laminated composites is the application of the "interacting mixture continua" developed by Bedford and Stern (1972) where the laminae are considered to be the interacting constituents. McNiven and Mengi (1979a,b,c) also employed the interacting mixture continua for modelling wave propagation in periodically structured composites. These two and the other approaches are tested by "the elasticity solution" (see, for example Sun et al. (1968a,b) which are later given (Sve, 1971) in a more general form.

Another important theoretical aspect of composite material has been indicated by Hashin (1983). In his outstanding review, Hashin has explicitly stated that ultimate understanding of the heterogeneous materials (not limited to, but also includes composite materials) requires a theory which takes into account the microstructure of the material, which in practical terms means a constitutive equation of nonlocal type. Beran and McCoy (1970) have already shown that the relation between the ensemble averages of stress and strain in a heterogeneous elastic solids is of the nonlocal form, i.e.

$$\sigma_{ij}(\underset{\sim}{x}) = \int_B L_{ijkl}(\underset{\sim}{x}, \underset{\sim}{x}')e_{kl}(\underset{\sim}{x}')\,\mathrm{d}x' \tag{1}$$

Recently Altan and Aifantis (1998) showed that the mixture theory of elasticity is equivalent to a special form of gradient dependent theory of elasticity under appropriate averaging process.

Nonlocal theory of elasticity which will be employed in this study has been developed by Eringen (1987) using thermomechanical arguments and by Eringen (1976b) by using variational principles (see also Eringen (1976b, 1978)). Although the nonlocal theory of elasticity seems more capable to represent the mechanical behavior of materials it suffers from theoretical and numerical difficulties inherent the Fredholm integral equation of the first kind. Probably that is why nonlocal elasticity has not been applied widely to various problems. Nevertheless, Eringen has shown that the wave propagation is dispersive (see Eringen and Edelen, 1972), and the stress field around a dislocation is finite Eringen (1976a) in nonlocal elasticity. Recently, Artan (1999) has shown that stress is finite in a half space loaded by a couple, and also shown that stress singularity is eliminated in punch problems (see Artan, 1996). To the best of our knowledge, Nowinski (1984, 1989, 1990) is the first researcher who has applied nonlocal elasticity to composite materials. In a previous paper of us (Altan and Artan, 2001) dispersion of horizontally polarized harmonic shear waves propagating in the layering direction in an infinite medium which consists of alternating layers of two isotropic nonlocal layers has been investigated. The subject of the presented paper is to investigate the dispersion of vertically polarized harmonic shear waves propagating in the layering direction in an infinite layered composite which consists of alternating layers of two isotropic nonlocal layers.

The next section contains a brief summary on the field equations of nonlocal elasticity. The problem considered in this study is introduced and formulated in the following section. The structure of the dispersion relation is criticized from the numerical point of view and some potential errors which may take place in the solution of them in both classical and nonlocal elasticity are indicated. Results are displayed in series of figures. The program Mathematica and Latex are used throughout.

2. The basic equations of nonlocal elasticity

The governing equations of the nonlocal theory of elasticity are

$$t_{kl,k} + \rho(f_l - \ddot{u}_l) = 0 \tag{2}$$

$$t_{kl} = \int_V \{\lambda'(|\underset{\sim}{x}' - \underset{\sim}{x}|)e_{jj}(\underset{\sim}{x}')\delta_{kl} + 2\mu'(|\underset{\sim}{x}' - \underset{\sim}{x}|)e_{kl}(\underset{\sim}{x}')\,\mathrm{d}v(\underset{\sim}{x}') \tag{3}$$

$$e_{kl}(\underset{\sim}{x}') = \frac{1}{2}(u_{k,l}(\underset{\sim}{x}') + u_{l,k}(\underset{\sim}{x}')) \tag{4}$$

where t_{kl} is the nonlocal stress tensor, u_k is the displacement vector, e_{kl} is the strain tensor and the comma as a subscript denotes the partial derivative, that is

$$t_{kl,m} = \frac{\partial t_{kl}}{\partial x_m}; \quad u'_{k,l} = \frac{\partial u'_k}{\partial x'_l} \tag{5}$$

we use the Einstein's summation convention for repeated indices. Eqs. (2) and (4) are the same, both in local and nonlocal elasticities. Eq. (3) express the fact that the stress at an arbitrary point $\underset{\sim}{x}$ depends on the strains at all the points $\underset{\sim}{x}'$ of the body. λ' and μ' are Lamé constants of the nonlocal medium and they depend on the distance between $\underset{\sim}{x}$ and $\underset{\sim}{x}'$. They can be taken as

$$\lambda' = \alpha(|\underset{\sim}{x}' - \underset{\sim}{x}|)\lambda; \quad \mu' = \alpha(|\underset{\sim}{x}' - \underset{\sim}{x}|)\mu \tag{6}$$

where λ and μ are the Lamé constants of the local case. $\alpha(|\underset{\sim}{x}' - \underset{\sim}{x}|)$ is called the kernel function and is the measure of the effect of the strain at $\underset{\sim}{x}'$ on the stress at $\underset{\sim}{x}$. The kernel function satisfies the following properties (Eringen, 1987, 1976a):

(i) $\alpha(|\underset{\sim}{x}'|)$ is a continuous function of $\underset{\sim}{x}'$, with a bounded support Ω, where $\alpha > 0$ inside the boundary $\partial\Omega$ of Ω, and $\alpha = 0$ outside

(ii)

$$\int_V \alpha(|\underset{\sim}{x}' - \underset{\sim}{x}|)\,\mathrm{d}v(\underset{\sim}{x}') = 1 \tag{7}$$

3. Propagation of SV waves in a laminated composite

Suppose that Lamé potentials in two media are given as follows

$$\phi_{(m)} = \Phi(x_2^m)\exp(\mathrm{i}(kx_1 - \omega t)) \tag{8}$$

$$\psi_{(m)} = \mathrm{i}\Psi(x_2^m)\exp(\mathrm{i}(kx_1 - \omega t)); \quad m = 1,2 \tag{9}$$

The functions $\phi_{(m)}$ and $\psi_{(m)}$ are the solutions of the differential equations (see Eringen, 1987)

$$c_1^{(m)^2}\nabla^2\phi_{(m)} - \left(1 - \frac{1}{\Omega^2}\nabla^2\right)\ddot{\phi}_{(m)} = 0 \tag{10}$$

$$c_2^{(m)^2}\nabla^2\psi_{(m)} - \left(1 - \frac{1}{\Omega^2}\nabla^2\right)\ddot{\psi}_{(m)} = 0 \tag{11}$$

where

$$c_1^{(m)} = \sqrt{\frac{\lambda_m + 2\mu_m}{\rho_m}}, \quad c_2^{(m)} = \sqrt{\frac{\mu_m}{\rho_m}} \tag{12}$$

Ω is nonlocality parameter. From the previous equations

$$\left(c_1^{(m)^2} - \frac{\omega^2}{\Omega^2}\right)\Phi'' + \left\{\omega^2\left(1 + \frac{k^2}{\Omega^2}\right) - c_1^{(m)^2}k^2\right\}\Phi = 0 \tag{13}$$

$$\left(c_2^{(m)^2} - \frac{\omega^2}{\Omega^2}\right)\Psi'' + \left\{\omega^2\left(1 + \frac{k^2}{\Omega^2}\right) - c_2^{(m)^2}k^2\right\}\Psi = 0 \tag{14}$$

If the solution of the above differential equations are substituted into Eqs. (8) and (9), Lamé potentials are obtained.

$$\phi_{(m)}(x_1, x_2^{(m)}, t) = \{A_m \sin(kp_m x_2^{(m)}) + B_m \cos(kp_m x_2^{(m)})\}\exp(\mathrm{i}(kx_1 - \omega t)) \tag{15}$$

$$\psi_{(m)}(x_1, x_2^{(m)}, t) = \mathrm{i}\{C_m \sin(kq_m x_2^{(m)}) + D_m \cos(kq_m x_2^{(m)})\}\exp(\mathrm{i}(kx_1 - \omega t)) \tag{16}$$

where

$$p_m = \sqrt{\frac{c^2}{c_1^{(m)^2} - (c^2k^2/\Omega^2)} - 1}; \quad q_m = \sqrt{\frac{c^2}{c_2^{(m)^2} - (c^2k^2/\Omega^2)} - 1}; \qquad c = \frac{\omega}{k} \tag{17}$$

Displacement field can be found as follows:

$$u_1^{(m)} = \frac{\partial\phi^{(m)}}{\partial x_1} + \frac{\partial\psi^{(m)}}{\partial x_2}; \quad u_2^{(m)} = \frac{\partial\phi^{(m)}}{\partial x_2} - \frac{\partial\psi^{(m)}}{\partial x_1} \tag{18}$$

by using Eq. (18)

$$\begin{aligned} u_1^{(m)}(x_1, x_2^{(m)}, t) = \mathrm{i}\mathrm{e}^{\mathrm{i}(kx_1 - \omega t)}k\Big\{&B_m \cos(kx_2^{(m)}p_m) + C_m \cos(kx_2^{(m)}q_m)q_m + A_m \sin(kx_2^{(m)}p_m) \\ &- D_m q_m \sin(kx_2^{(m)}q_m)\Big\} \end{aligned} \tag{19}$$

$$\begin{aligned} u_2^{(m)}(x_1, x_2^{(m)}, t) = \mathrm{e}^{\mathrm{i}(kx_1 - \omega t)}k\Big\{&D_m \cos(kx_2^{(m)}q_m) + A_m \cos(kx_2^{(m)}p_m)p_m - B_m p_m \sin(kx_2^{(m)}p_m) \\ &+ C_m \sin(kx_2^{(m)}q_m)\Big\} \end{aligned} \tag{20}$$

The kernel function is chosen as

$$\alpha(|\underset{\sim}{x} - \underset{\sim}{x}'|) = \frac{\Omega^2}{2\pi} K_0\left(\Omega\sqrt{(x_1 - x_1')^2 + (x_2 - x_2')^2}\right) \tag{21}$$

where $K_0(x)$ is modified Bessel function of the second kind (see Fig. 1). A close inspection of the non-local kernel shows that this function dies out very fast. In other words, strains in an immediate vicinity of the point contribute to the stress at this point, strains outside this region do not. "Therefore, ignoring the effect of non-homogeneous character of the elastic constants on the local stress will not cause a significant error, especially at the points far enough from the interface."

After tedious calculations the following result is obtained.

$$\frac{\Omega^2}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K_0\left(\sqrt{\Omega\{(x_1 - x_1')^2 + (x_2 - x_2')^2\}}\right) f(x_1', x_2')\,\mathrm{d}x_1'\,\mathrm{d}x_2' = \frac{f(x_1, x_2)}{1 + (k/\Omega^2)(1 + p^2)} \tag{22}$$

where

$$f(x_1, x_2) = \begin{Bmatrix} \sin(kpx_2) \\ \cos(kpx_2) \end{Bmatrix} \times \begin{Bmatrix} \sin(kx_1) \\ \cos(kx_1) \end{Bmatrix} \tag{23}$$

The nonlocal stress field can be obtained by using Eqs. (3), (6) and (22)

$$\begin{aligned} t_{12}^{(m)} = -\mathrm{i}e^{\mathrm{i}(kx_1 - \omega t)} k^2 \mu_m \Bigg\{ & \frac{-2A_m \cos(kx_2^{(m)} p_m) p_m}{1 + (k/\Omega)^2(1 + p_m^2)} + \frac{D_m \cos(kx_2^{(m)} q_m)(q_m^2 - 1)}{1 + (k/\Omega)^2(1 + q_m^2)} + \frac{2B_m p_m \sin(kx_2^{(m)} p_m)}{1 + (k/\Omega)^2(1 + p_m^2)} \\ & + \frac{C_m (q_m^2 - 1)\sin(kx_2^{(m)} q_m)}{1 + (k/\Omega)^2(1 + q_m^2)} \Bigg\} \end{aligned} \tag{24}$$

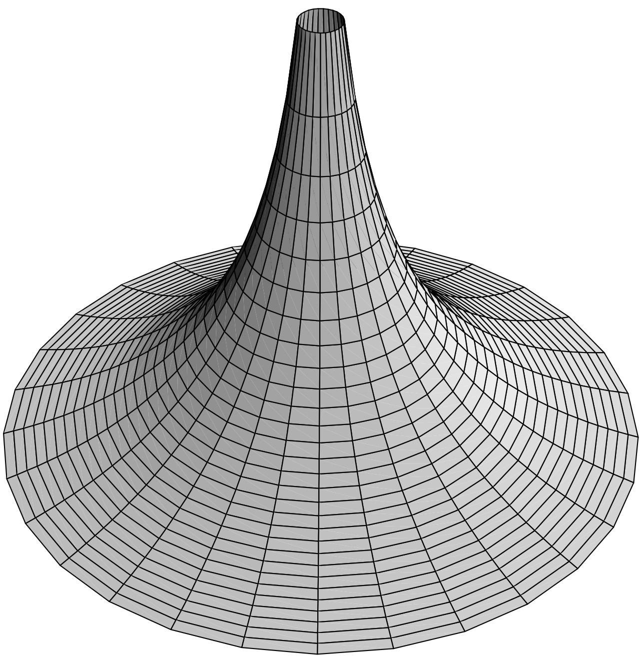

Fig. 1. Kernel function.

$$t_{22}^{(m)} = -e^{i(kx-\omega t)}k^2\left\{\frac{B_m\cos(kx_2^{(m)}p_m)(\lambda_m+(\lambda_m+2\mu_m)p_m^2)}{1+(k/\Omega)^2(1+p_m^2)} - \frac{2\mu_m C_m\cos(kx_2^{(m)}q_m)q_m}{1+(k/\Omega)^2(1+q_m^2)}\right.$$
$$\left. + \frac{A_m(\lambda_m+(\lambda_m+2\mu_m)p_m^2)\sin(kx_2^{(m)}p_m)}{1+(k/\Omega)^2(1+p_m^2)} + \frac{2\mu_m D_m q_m\sin(kx_2^{(m)}q_m)}{1+(k/\Omega)^2(1+q_m^2)}\right\} \tag{25}$$

The jump conditions at the interface are (see Fig. 2)

$$\begin{aligned} &u_1^{(1)}(h_1/2) = u_1^2(-h_2/2); \quad u_2^{(1)}(h_1/2) = u_2^2(-h_2/2); \\ &t_{12}^{(1)}(h_1/2) = t_{12}^{(2)}(-h_2/2); \quad t_{22}^{(1)}(h_1/2) = t_{12}^{(2)}(-h_2/2) \end{aligned} \tag{26}$$

3.1. Anti-symmetrical deformations with respect to the midplanes of the layers

$B_m = C_m = 0$. In this case, the longitudinal displacements are odd and the transverse displacements are even. The four conditions of the type Eq. (26) yield four homogeneous equations for the constants A_1, A_2, D_1, D_2. The requirement that the determinant of the coefficients must vanish yields the dispersion relation in the form.

$$g(\xi,\beta,\eta,\gamma,v_1,v_2,\theta,k,\Omega) = \det(m) = 0 \tag{27}$$

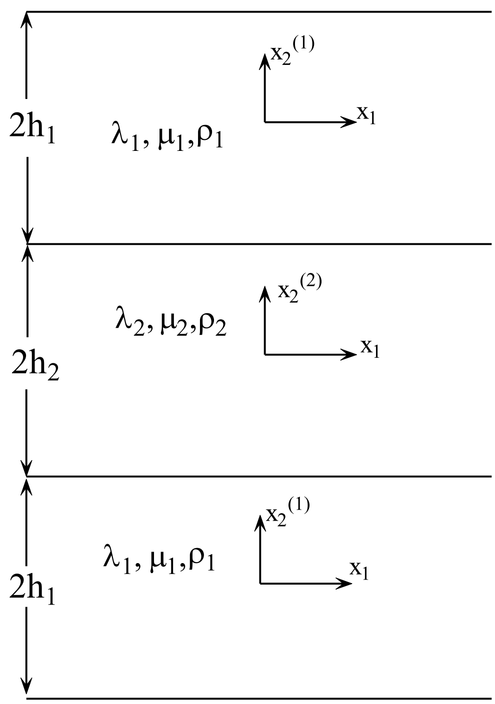

Fig. 2. Laminated composite and coordinate axis.

where

$$\xi = kh_1, \quad \eta = \frac{h_1}{h_2}, \quad \gamma = \frac{\mu_1}{\mu_2}, \quad \theta = \frac{\rho_1}{\rho_2}, \quad \beta = \frac{c}{\sqrt{\mu_2/\rho_2}} \tag{28}$$

$$p_1 = \sqrt{\frac{1}{\frac{2\gamma(v_1-1)}{\beta^2\theta(2v_1-1)} - \left(\frac{k}{\Omega}\right)^2} - 1}; \quad p_2 = \sqrt{\frac{1}{\frac{2(v_2-1)}{\beta^2(2v_2-1)} - \left(\frac{k}{\Omega}\right)^2} - 1} \tag{29}$$

$$q_1 = \sqrt{\frac{1}{\frac{\gamma}{\beta^2\theta} - \left(\frac{k}{\Omega}\right)^2} - 1}; \quad q_2 = \sqrt{\frac{1}{\frac{1}{\beta^2} - \left(\frac{k}{\Omega}\right)^2} - 1} \tag{30}$$

$$m_{11} = \sin\left(\frac{\xi p_1}{2}\right) \tag{31}$$

$$m_{12} = \sin\left(\frac{\xi p_2}{2\eta}\right) \tag{32}$$

$$m_{13} = -q_1 \sin\left(\frac{\xi q_1}{2}\right) \tag{33}$$

$$m_{14} = -q_2 \sin\left(\frac{\xi q_2}{2\eta}\right) \tag{34}$$

$$m_{21} = p_1 \cos\left(\frac{\xi p_1}{2}\right) \tag{35}$$

$$m_{22} = -p_2 \cos\left(\frac{\xi p_2}{2\eta}\right) \tag{36}$$

$$m_{23} = \cos\left(\frac{\xi q_1}{2}\right) \tag{37}$$

$$m_{24} = -\cos\left(\frac{\xi q_2}{2\eta}\right) \tag{38}$$

$$m_{31} = \frac{1}{(v_1-1)\Omega^2}\left(p_1(k^2\beta^2\theta(2v_1-1) - 2\gamma(v_1-1)\Omega^2)\cos\left(\frac{\xi p_1}{2}\right)\right) \tag{39}$$

$$m_{32} = -\frac{1}{(v_2-1)\Omega^2}\left(p_2(k^2\beta^2(2v_2-1) - 2(v_2-1)\Omega^2)\cos\left(\frac{\xi p_2}{2\eta}\right)\right) \tag{40}$$

$$m_{33} = \frac{1}{\Omega^2}\left(-2\gamma\Omega^2 + \beta^2\theta(2k^2+\Omega^2)\right)\cos\left(\frac{\xi q_1}{2}\right) \tag{41}$$

$$m_{34} = -\frac{1}{\Omega^2}(2k^2\beta^2 + (\beta^2-2)\Omega^2)\cos\left(\frac{\xi q_2}{2\eta}\right) \tag{42}$$

$$m_{41} = \frac{1}{(v_1 - 1)\Omega^2}\left((k^2\beta^2\theta(2v_1 - 1) + (\beta^2\theta - 2\gamma)(v_1 - 1)\Omega^2)\sin\left(\frac{\xi p_1}{2}\right)\right) \quad (43)$$

$$m_{42} = \frac{1}{(v_2 - 1)\Omega^2}\left((k^2\beta^2(2v_2 - 1) + (\beta^2 - 2)(v_2 - 1)\Omega^2)\sin\left(\frac{\xi p_2}{2\eta}\right)\right) \quad (44)$$

$$m_{43} = \frac{2q_1}{\Omega^2}(\gamma\Omega^2 - k^2\beta^2\theta)\sin\left(\frac{\xi q_1}{2}\right) \quad (45)$$

$$m_{44} = \frac{2q_2}{\Omega^2}(\Omega^2 - k^2\beta^2)\sin\left(\frac{\xi q_2}{2\eta}\right) \quad (46)$$

The domain of the function $g(\xi, \beta, \eta, \gamma, v_1, v_2, \theta, k, \Omega)$ for various values of β is given below.

$$\begin{aligned} &g(\xi, \beta, 4, 50, 0.3, 0.35, 3, 2, 50);\ 7.6315 < \beta < 25 \\ &g(\xi, \beta, 4, 100, 0.3, 0.35, 3, 2, 50);\ 10.7926 < \beta < 25 \\ &g(\xi, \beta, 4, 50, 0.3, 0.35, 3, 2, 100);\ 7.6361 < \beta < 50 \\ &g(\xi, \beta, 4, 50, 0.3, 0.35, 3, 2, 50);\ 10.7991 < \beta < 50 \end{aligned} \quad (47)$$

There are dispersion curves in the intersection of the surfaces $g(\xi, \beta, \eta, \gamma, v_1, v_2, \theta, k, \Omega)$ and $z = 0$ plane (see Figs. 3–10).

3.2. Symmetrical deformations with respect to the midplanes of the layers

$A_m = D_m = 0$. In this case, the longitudinal displacements are even and the transverse displacements are odd. The four conditions of the type Eq. (26) yield four homogeneous equations for the constants B_1, B_2, C_1, C_2. The requirement that the determinant of the coefficients must vanish yields the dispersion relation in the form.

$$f(\xi, \beta, \eta, \gamma, v_1, v_2, \theta, k, \Omega) = \det(n) = 0 \quad (48)$$

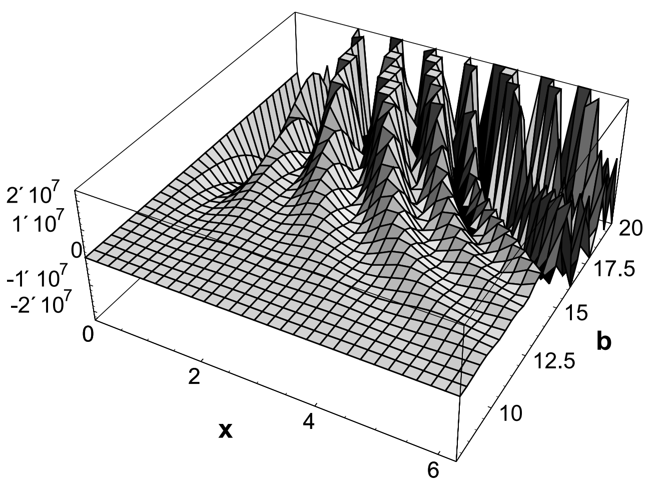

Fig. 3. Surface of $g(\xi, \beta, 4, 50, 0.3, 0.35, 3, 2, 50)$.

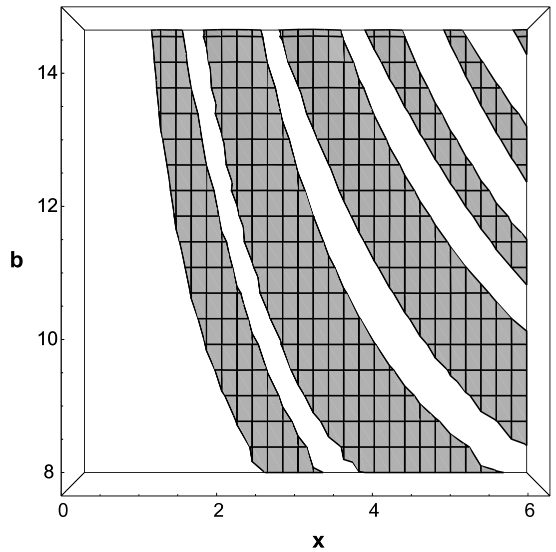

Fig. 4. Dispersion curves for $g(\xi, \beta, 4, 50, 0.3, 0.35, 3, 2, 50) = 0$.

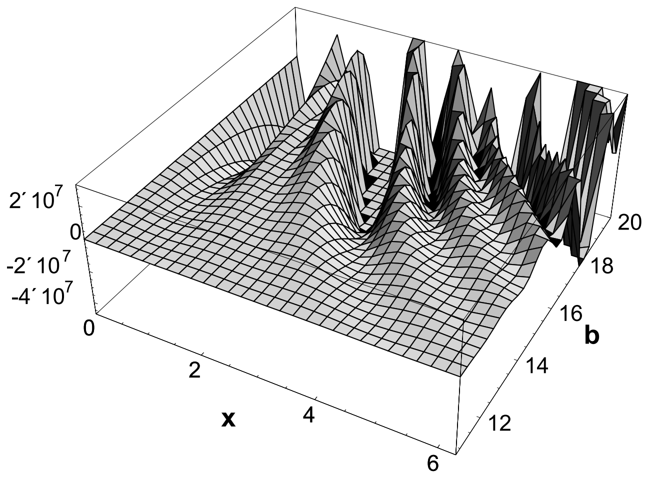

Fig. 5. Surface of $g(\xi, \beta, 4, 100, 0.3, 0.35, 3, 2, 50)$.

where

$$n_{11} = \cos\left(\frac{\xi p_1}{2}\right) \tag{49}$$

$$n_{12} = -\cos\left(\frac{\xi p_2}{2\eta}\right) \tag{50}$$

$$n_{13} = q_1 \cos\left(\frac{\xi q_1}{2}\right) \tag{51}$$

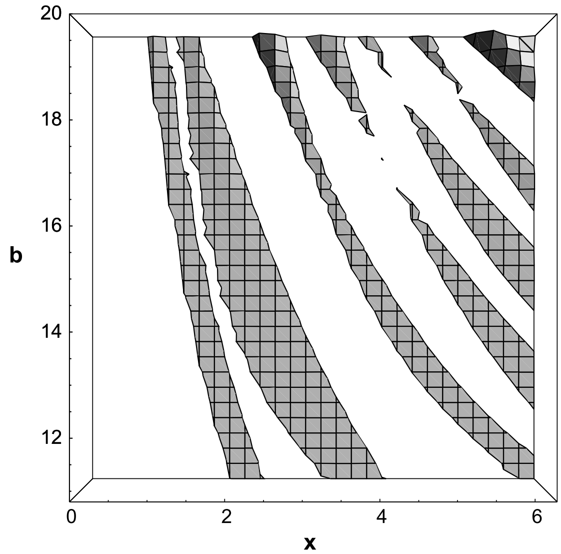

Fig. 6. Dispersion curves for $g(\xi, \beta, 4, 100, 0.3, 0.35, 3, 2, 50) = 0$.

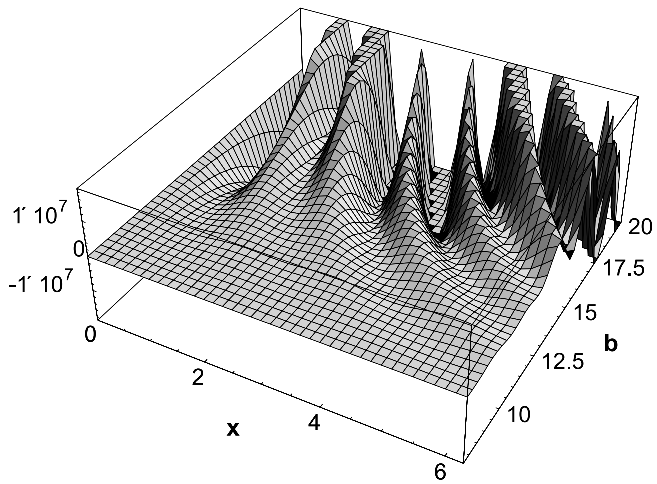

Fig. 7. Surface of $g(\xi, \beta, 4, 50, 0.3, 0.35, 3, 2, 100)$.

$$n_{14} = -q_2 \cos\left(\frac{\xi q_2}{2\eta}\right) \tag{52}$$

$$n_{21} = -p_1 \sin\left(\frac{\xi p_1}{2}\right) \tag{53}$$

$$n_{22} = -p_2 \sin\left(\frac{\xi p_2}{2\eta}\right) \tag{54}$$

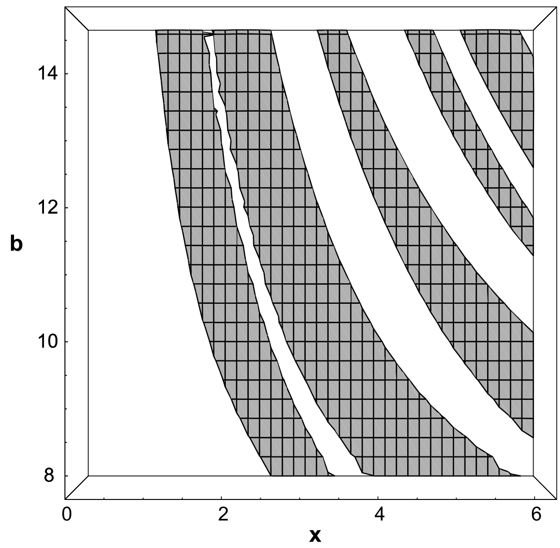

Fig. 8. Dispersion curves for $g(\xi, \beta, 4, 50, 0.3, 0.35, 3, 2, 100) = 0$.

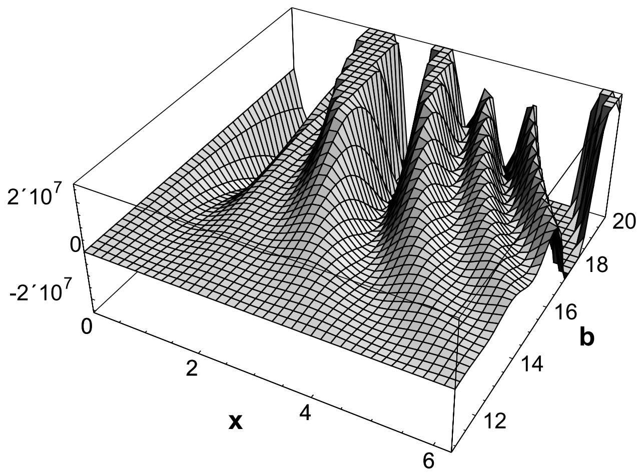

Fig. 9. Surface of $g(\xi, \beta, 4, 100, 0.3, 0.35, 3, 2, 100)$.

$$n_{23} = \sin\left(\frac{\xi q_1}{2}\right) \tag{55}$$

$$n_{24} = \sin\left(\frac{\xi q_2}{2\eta}\right) \tag{56}$$

$$n_{31} = \frac{1}{(v_1 - 1)\Omega^2}\left(p_1(k^2\beta^2\theta(2v_1 - 1) - 2\gamma(v_1 - 1)\Omega^2)\sin\left(\frac{\xi p_1}{2}\right)\right) \tag{57}$$

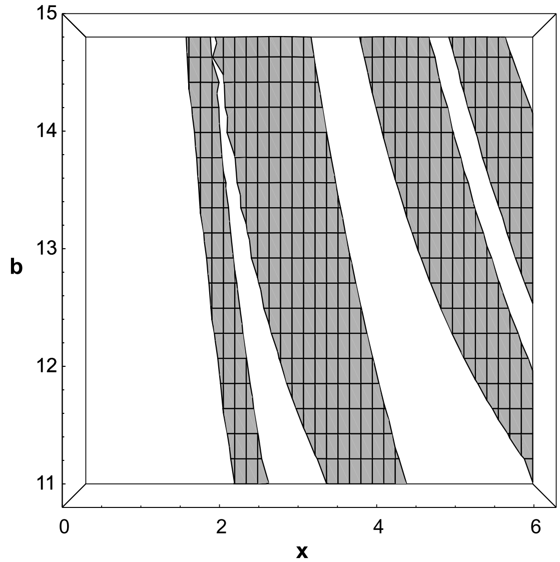

Fig. 10. Dispersion curves for $g(\xi, \beta, 4, 100, 0.3, 0.35, 3, 2, 100) = 0$.

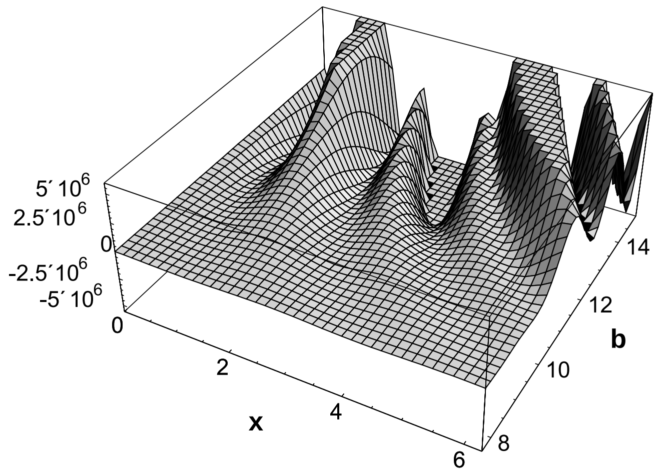

Fig. 11. Surface of $f(\xi, \beta, 4, 50, 0.3, 0.35, 3, 2, 50)$.

$$n_{32} = -\frac{1}{(v_2 - 1)\Omega^2}\left(p_2(k^2\beta^2(2v_2 - 1) - 2(v_2 - 1)\Omega^2)\sin\left(\frac{\xi p_2}{2\eta}\right)\right) \tag{58}$$

$$n_{33} = \frac{1}{\Omega^2}\left(-2\gamma\Omega^2 + \beta^2\theta(2k^2 + \Omega^2)\right)\sin\left(\frac{\xi q_1}{2}\right) \tag{59}$$

$$n_{34} = \frac{1}{\Omega^2}(2k^2\beta^2 + (\beta^2 - 2)\Omega^2)\sin\left(\frac{\xi q_2}{2\eta}\right) \tag{60}$$

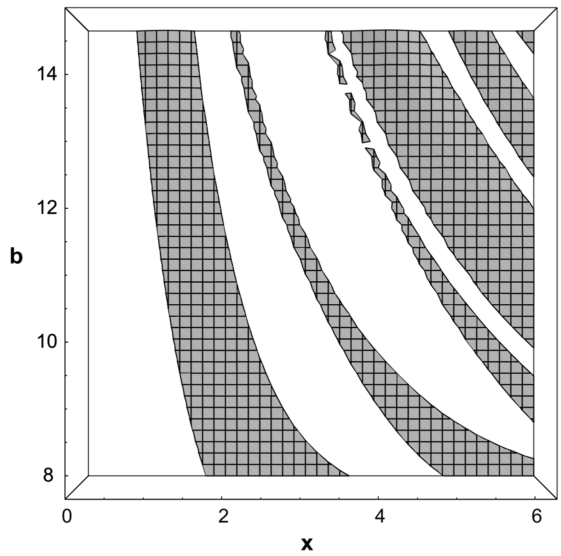

Fig. 12. Dispersion curves for $f(\xi, \beta, 4, 50, 0.3, 0.35, 3, 2, 50) = 0$.

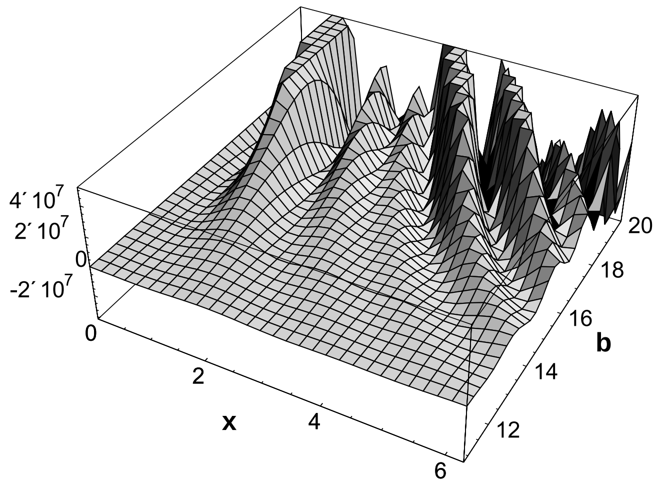

Fig. 13. Surface of $f(\xi, \beta, 4, 100, 0.3, 0.35, 3, 2, 50)$.

$$n_{41} = \frac{1}{(v_1 - 1)\Omega^2}\left((k^2\beta^2\theta(2v_1 - 1) + (\beta^2\theta - 2\gamma)(v_1 - 1)\Omega^2)\cos\left(\frac{\xi p_1}{2}\right)\right) \tag{61}$$

$$n_{42} = \frac{1}{(v_2 - 1)\Omega^2}\left((k^2\beta^2(1 - 2v_2) - (\beta^2 - 2)(v_2 - 1)\Omega^2)\cos\left(\frac{\xi p_2}{2\eta}\right)\right) \tag{62}$$

$$n_{43} = \frac{2q_1}{\Omega^2}(k^2\beta^2\theta - \gamma\Omega^2)\cos\left(\frac{\xi q_1}{2}\right) \tag{63}$$

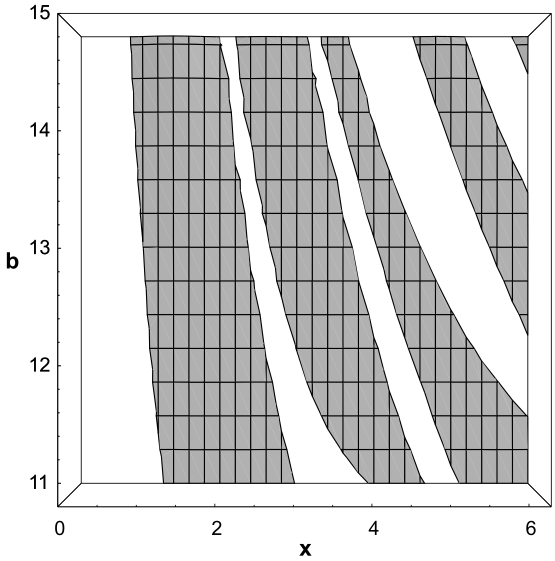

Fig. 14. Dispersion curves for $f(\xi, \beta, 4, 100, 0.3, 0.35, 3, 2, 50) = 0$.

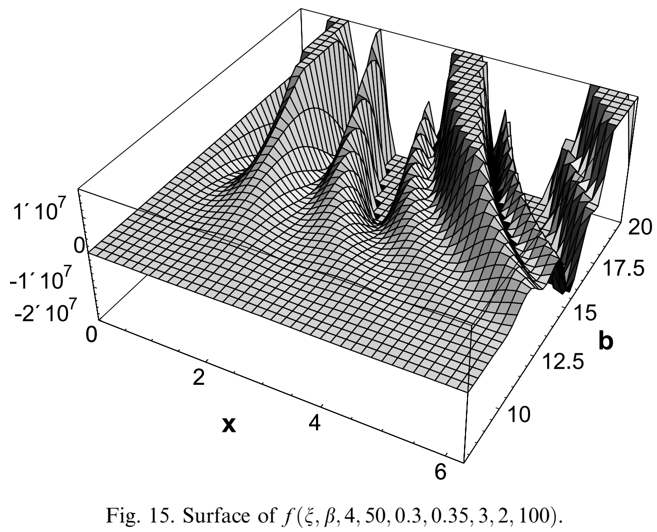

Fig. 15. Surface of $f(\xi, \beta, 4, 50, 0.3, 0.35, 3, 2, 100)$.

$$n_{44} = \frac{2q_2}{\Omega^2}(\Omega^2 - k^2\beta^2)\cos\left(\frac{\xi q_2}{2\eta}\right) \tag{64}$$

The function $f(\xi, \beta, \eta, \gamma, v_1, v_2, \theta, k, \Omega)$ has the same domain with the function $g(\xi, \beta, \eta, \gamma, v_1, v_2, \theta, k, \Omega)$. There are dispersion curves in the intersection of the surfaces $f(\xi, \beta, \eta, \gamma, v_1, v_2, \theta, k, \Omega)$ and $z = 0$ plane (see Figs. 11–18).

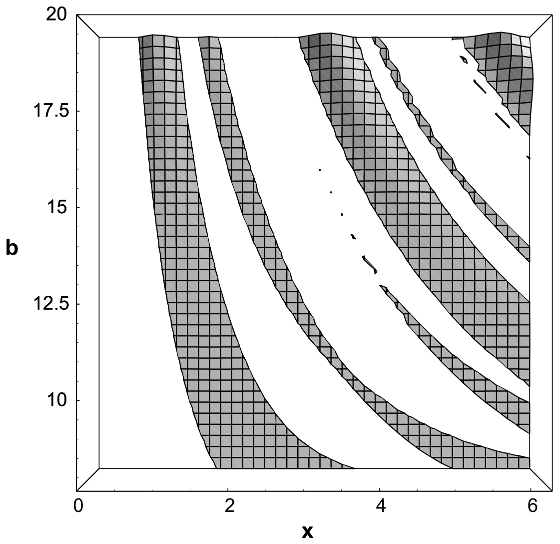

Fig. 16. Dispersion curves for $f(\xi,\beta,4,50,0.3,0.35,3,2,100)=0$.

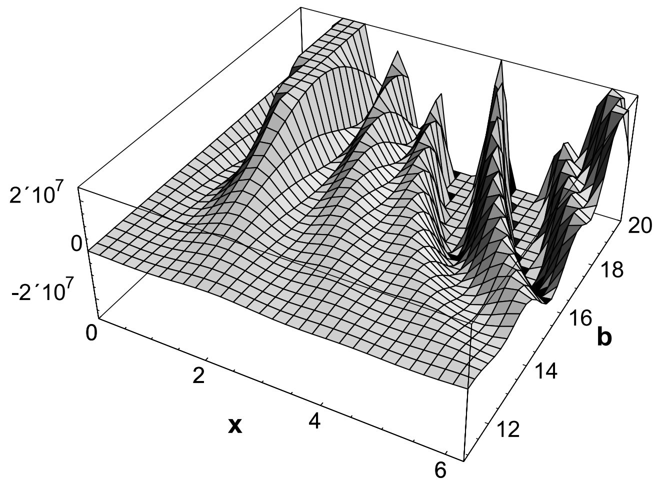

Fig. 17. Surface of $f(\xi,\beta,4,100,0.3,0.35,3,2,100)$.

4. Conclusion

In this paper, the dispersion relation for a vertically harmonic shear wave propagating in the layering direction in an infinite medium which consists of alternating layers of two homogeneous, isotropic materials is investigated within the framework of nonlocal elasticity. Symmetric and anti-symmetric cases are studied separately. It has been shown that the numerical structure of the problem under investigation can be tricky, may cause misleading results. The effect of nonlocality on the dispersion relation for the propagation of vertically polarized shear waves is shown.

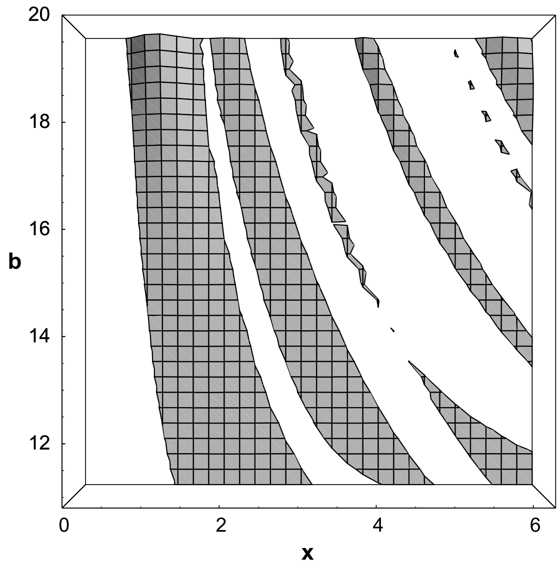

Fig. 18. Dispersion curves for $f(\xi, \beta, 4, 100, 0.3, 0.35, 3, 2, 100) = 0$.

The dispersion relation given by Eq. (27) is of transandant equation, and therefore it can be solved only by approximate methods. It is also known that all algorithms based on approximate method for solving transandant equation employ an initial guess. On the other hand, another well-known fact is that the dispersion relation (27) has many solutions, more precisely, there are infinitely many frequencies corresponding to a certain wave number (modes). While analyzing the dispersion relation (27) by common techniques the following drawbacks are observed.

(a) Some modes can be easily disregarded: Initial guess employed at the beginning of the algorithm may cause to jump to not to the next mode but to another mode.
(b) It can be picked some points from the next mode when calculating the points on a certain mode, especially if the modes are close each other. It was not possible to remedy this situation by decreasing the absolute or relative error in calculating the roots.

In order to eliminate these drawback the surfaces defined by Eqs. (27) and (48) are intersected by $z = 0$ plane. The overall picture of the surfaces given by Eqs. (27) and (48) provides a global view and with this information at hand the possibility of making mistakes caused by the drawback mentioned above can be reduced to a reasonable level. During the course of this study, it has been observed that the dispersion curves can be too close to each other for some values of parameters of the problem. This situation arises more frequently in the nonlocal solution of the problem.

The dispersion and attenuation of waves has a prime role in analyzing the dynamic behaviour of composite materials. In analyzing the phenomena, such as sound isolation, impact behaviour, ultrasonic testing, etc. wave propagation has a central importance. Although wave propagation in nonlocal elasticity has been analyzed by Eringen and Edelen (1972), and Eringen (1987). Ari (1982) applications of nonlocal elasticity to composite materials is quite limited. Nowinski studied the propagation of Love waves (Nowinski, 1984), transmission of wave across the interface of two dissimilar elastic half spaces Nowinski (1989), and propagation of waves in an elastic multilayer periodic media (Nowinski, 1989). The presented study can be considered as the continuation of the studies on periodic layered composites started by in the direction of nonlocality. As is also indicated above, nonlocality may exist in a material due to crystal structure, grains, impurities, dislocations, micro-cracks, etc. Therefore, the nonlocal effects may become important especially for the propagation of high frequency waves.

The following properties are observed in the nonlocal solution of the problem.

(a) The characteristics of the dispersion relation have a strong dependence on the nonlocal parameter. The pattern of the dispersion curves, location of modes, stop bands, etc. dramatically change for decreasing values of the nonlocal parameter.
(b) The stress field reverts to the classical counterpart if the nonlocality parameter Ω goes to infinity (see Sun et al., 1968b).
(c) The dispersion curves are getting closer as the nonlocality parameter Ω as well as the ratio of shear modulus γ increase.

The properties observed in the dispersion relations for a vertically polarized harmonic shear wave propagating in the direction of layering show that nonlocal elasticity is more capable in representing the dynamic behaviour of layered composites.

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